Complex Numbers

Question1

If Z_1 and Z_2 are two non-zero complex numbers, then which of the following is not true?

KCET 2025

Options:

A. $\overline{Z_1 + Z_2} = \bar{Z}_1 + \bar{Z}_2$

B. $|Z_1 Z_2| = |Z_1| \cdot |Z_2|$

C. $\overline{\mathrm{Z}_1 \mathrm{Z}_2} = \overline{\mathrm{Z}}_1 \cdot \overline{\mathrm{Z}}_2$

D. $|\mathrm{Z}_1 + \mathrm{Z}_2| \geq |\mathrm{Z}_1| + |\mathrm{Z}_2|$

Answer: D

Solution:

$|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

Question2

The real value of ' α ' for which $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely real is

KCET 2024

Options:

A. $(n+1)\frac{\pi}{2}, n \in N$

B. $(2n+1)\frac{\pi}{2}, n \in N$

C. $n\pi, n \in N$

D. $(2n-1)\frac{\pi}{2}, n \in N$

Answer: C

Solution:

Given, $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely real

i.e. $\frac{1-i\sin\alpha}{1+2i\sin\alpha} \times \frac{(1-2i\sin\alpha)}{(1-2i\sin\alpha)}$

$$= \frac{1-i\sin\alpha-2i\sin\alpha+2i^2\sin^2\alpha}{1-4i^2\sin^2\alpha}$$
$$= \frac{1-3i\sin\alpha-2\sin^2\alpha}{1+4\sin^2\alpha}$$
$$= \frac{1-2\sin^2\alpha}{1+4\sin^2\alpha} + i\left(\frac{-3\sin\alpha}{1+4\sin^2\alpha}\right)$$

which is given to purely real

$\Rightarrow \quad \frac{-3\sin\alpha}{1+4\sin^2\alpha} = 0$

$\Rightarrow \quad -3\sin\alpha = 0 \Rightarrow \sin\alpha = 0$

Hence, $\alpha = n\pi, n \in N$

Question3

The modulus of the complex number $\frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is

KCET 2023

Options:

A. $\frac{2}{\sqrt{2}}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{2}}{4}$

D. $\frac{4}{\sqrt{2}}$

Answer: C

Solution:

$$\text{Here, } \frac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$$
$$=\frac{(1+i^2+2i)(1+3i)}{4-4i-12i+12i^2}$$
$$=\frac{2i+6i^2}{-8-16i}=\frac{2i-6}{-8-16i}$$
$$=\frac{i-3}{-4-8i}$$
$$=\frac{3-i}{(4+8i)}\times\frac{(4-8i)}{(4-8i)}$$
$$=\frac{12-28i-8}{16+64}$$
$$=\frac{4-28i}{80}=\frac{1-7i}{20}$$
$$=\frac{1}{20}-\frac{7i}{20}$$

Let $z=x+iy$

$\because |z|=\sqrt{x^2+y^2}$

$\Rightarrow |z|=\sqrt{\left(\frac{1}{20}\right)^2+\left(\frac{7}{20}\right)^2}$

$\Rightarrow |z|=\frac{1}{20}\sqrt{50}=\frac{5}{20}\sqrt{2}$

$\Rightarrow |z|=\frac{\sqrt{2}}{4}$

Question4

If $3x+i(4x-y)=6-i$ where x and y are real numbers, then the values of x and y are respectively,

KCET 2022

Options:

A. 3, 9

B. 2, 4

C. 2, 9

D. 3, 4

Answer: C

Solution:

$3x + i(4x - y) = 6 - i$

On comparing real and imaginary parts of LHS and RHS, we get

$$\begin{aligned} & 3x = 6 \text{ and } 4x - y = -1 \\ \Rightarrow \quad & 3x = 6 \Rightarrow x = 2 \end{aligned}$$

Put $x = 2$ into $4x - y = -1$, we get

$$\begin{aligned} & 4 \times 2 - y = -1 \\ \Rightarrow \quad & 8 + 1 = y \Rightarrow y = 9 \end{aligned}$$

Question5

If $\left(\frac{1+i}{1-i}\right)^{x} = 1$, then

KCET 2021

Options:

A. $x = 4n + 1, n \in N$

B. $x = 2n + 1, n \in N$

C. $x = 2n, n \in N$

D. $x = 4n, n \in N$

Answer: D

Solution:

Given, $\left(\frac{1+i}{1-i}\right)^x = 1$

$\Rightarrow \quad \left[\frac{(1+i)}{1-i)} \times \frac{(1+i)}{(1+i)}\right]^x = 1$

$\Rightarrow \quad \left[\frac{1+i^2+2i}{1^2-i^2}\right]^x = 1$

$\Rightarrow \quad \left[\frac{1-1+2i}{1+1}\right]^x = 1$

$\Rightarrow \quad i^x = 1$

$\Rightarrow \quad i^x = 1 = i^{4n}$ where $n \in N$

$\therefore x = 4n, n \in N.$

Question6

If $z = x + iy$, then the equation $|z+1| = |z-1|$ represents

KCET 2020

Options:

A. a circle

B. a parabola

C. X-axis

D. Y-axis

Answer: D

Solution:

Given,

$$z = x + iy$$
$$|z+1| = |z-1|$$
$$|z+1|^2 = |z-1|^2$$
$$|x+iy+1|^2 = |x+iy-1|^2$$
$$(x+1)^2 + y^2 = (x-1)^2 + y^2$$
$$(x+1)^2 - (x-1)^2 = 0$$
$$(x+1-x+1)(x+1+x-1) = 0$$
$$x = 0$$
$$\therefore |z+1| = |z-1| \text{ represents } Y\text{-axis.}$$

Question7

If $\left(\frac{1-i}{1+i}\right)^{96} = a + ib$, then (a, b) is

KCET 2018

Options:

A. $(1,1)$

B. $(0,1)$

C. $(1,0)$

D. $(0,-1)$

Answer: B

Solution:

To solve the given expression $\left(\frac{1-i}{1+i}\right)^{96} = a + ib$, follow these steps:

Simplify the Fraction:

$\frac{1-i}{1+i}$

Multiply the numerator and the denominator by the conjugate of the denominator:

$\frac{(1-i)(1-i)}{(1+i)(1-i)}$

Calculate the Numerator and Denominator:

$(1-i)(1-i) = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$

$(1+i)(1-i) = 1 - i^2 = 1 - (-1) = 2$

Simplified Expression:

$\frac{-2i}{2} = -i$

Raise to the Power of 96:

$(-i)^{96}$

Understanding Powers of $-i$:

Since $(-i)^4 = 1$, calculate $(-\mathrm{i}))^{96}$:

$(-i)^{96} = ((-i)^4)^{24} = 1^{24} = 1$

Thus, the expression simplifies to:

$a + ib = 1 + 0i$

Result:

Therefore, $(a, b) = (1, 0)$.

Question8

If $\left(\frac{1+i}{1-i}\right)^m = 1$, then the least positive integral value of m is

KCET 2017

Options:

A. 2

B. 4

C. 3

D. 1

Answer: B

Solution:

We have,

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left[\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right]^m = 1$$

$$\Rightarrow \left[\frac{1+2i+i^2}{1-i^2}\right]^m = 1$$

$$\Rightarrow \left[\frac{1+2i-1}{1-(-1)}\right]^m = 1 \qquad [\because i^2 = -1]$$

$$\Rightarrow \left[\frac{2i}{2}\right]^m = 1$$

$$\Rightarrow i^m = 1$$

$$\Rightarrow i^m = 4 \qquad [\because 1^4 = 1]$$

$$\Rightarrow m = 4$$
